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Technical Note

Gabor based analysis prior formulation for EEG signal reconstruction

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ABSTRACT

This paper deals with the problem of tele-monitoring EEG signals. In EEG tele-monitoring system, the integral step is to compress the signals in computationally efficient manner so that they can be transmitted over a limited bandwidth. In such a situation a Compressed Sensing (CS) framework for compressing and recovering the signals is the most viable approach. Previously the well known synthesis prior formulation is used for reconstruction. For the first time in this work, we show that the lesser known analysis prior formulation is a more appropriate way to frame the reconstruction problem. We show that our method yields better results than the previous synthesis prior formulation.

1. Introduction

Recently Wireless Body Area Networks (WBAN) are being used for monitoring the health of individuals remotely (base station). Various types of health indicating signals such as EEG, ECG, Pulse etc. can be monitored thus. In this work, we are particularly interested in the acquisition and transmission of EEG signals over WBAN. EEG signals are useful for detecting epileptic seizures and strokes.

The main problem in any wireless sensor network (including WBAN) is its limited energy resources. The main computational power for any sensor network is expended in sampling, processing and transmitting. There is not much scope in reducing the power consumption associated with sampling. The processing and the transmission costs can be reduced by designing a cheap encoder for compressing the sampled signal.

In traditional transform coding (for compression), the encoder is complex and the decoder is simple. However in this scenario the requirement is exactly the opposite. The encoder needs to be simple (less power hungry) while the decoder is at the base station where there is no premium on computational resources.

Compressed Sensing (CS) based encoder–decoder is ideal for this scenario. CS uses random projections (from higher to lower dimensions) for encoding; this operation is computationally cheap (reduces processing power). Since the signal is now compressed, the transmission power reduces as well. However, the decoding is computationally expensive and requires solving a non-linear optimization problem. However, as mentioned before, this poses no issue since there are powerful computers at the base station.

There have been a few studies that propose compressed sensing based compression and reconstruction schemes for EEG [1–7]. However each of them suffer from certain shortcomings. It is not possible to fully understand these shortcomings without some understanding of CS. Therefore we will briefly discuss CS in Section 2. The main contribution of the prior works and their shortcomings will be discussed in Section 3. The proposed formulation is described in Section 4. We discuss the experimental results in Section 5. Finally the conclusions of this work and scope of further research is discussed in Section 6.

2. Compressed sensing

Compressed sensing (CS) studies the problem of solving an under-determined systems of linear equations where the solution is known to be sparse,

$$y_{m\times 1} = A_{m\times n}x_{n\times 1}, \quad m < n \tag{1}$$

In general (1) has infinitely many solutions. In CS, one is interested in finding a sparse solution to (1), i.e. even though x is of size n, it is s-sparse (s non-zero values the rest $n-s$ are all zeroes). It has been proven that for most cases, the sparse solution is unique [8]; thus one may as well seek the sparsest solution (owing to its uniqueness) 'Seeking the sparsest solution' is mathematically represented in the following form,

$$\min_x ||x||_0 \quad \text{subject to} \quad y = Ax \tag{2}$$

Here the l_0-norm counts the number of non-zeroes in x.

Unfortunately, solving (2) is an NP hard problem [9], and any attempt to solve it is as good as brute force search. Thus it is not feasible to solve the l_0-norm minimization problem for any practical large-scale system.

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Theoretical studies in CS shows that it is possible to surrogate the NP hard l_0-norm minimization problem by replacing it with its nearest convex surrogate the l_1-norm; under certain conditions the result obtained from solving the convex optimization problem (l_1-norm minimization) is guaranteed to be the sparsest solution [8,10]. Thus CS advocates recovery of the sparsest solution by solving the following,

$$\min_x ||x||_1 \quad \text{subject to} \quad y = Ax \tag{3}$$

This is a convex optimization problem and can be solved by linear programming. Solving (3) is a topic in itself, and over the years researchers in optimization have developed fast and efficient solvers for large scale problems of the form (3).

Till now we have been speaking about the theoretical problem of solving an under-determined linear system of equations. In applied Compressed Sensing, the solution 'x' to be recovered is the 'signal' of interest, A is the 'measurement operator' and y is the collected 'measurements'. From now on we will be using to these terms.

Natural signals are almost never sparse but are compressible in a transform domain, e.g. images are sparse in Discrete Cosine Transform (DCT) or wavelet basis, speech is sparse in Short Time Fourier Transform, Seismic waves are sparse in Curvelet basis, EEG is sparse in Gabor basis and so on. Transforms which are orthogonal or are tight-framed are convenient for the CS community

$$\begin{aligned} &\text{Orthogonal}: \quad S^T S = I = SS^T \\ &\text{Tight-frame}: \quad S^T S = I \neq SS^T \end{aligned} \tag{4}$$

For both orthogonal and tight-framed transforms, the analysis–synthesis equations are expressed as follows,

$$\begin{aligned} &\text{analysis}: \quad \alpha = Sx \\ &\text{synthesis}: \quad x = S^T\alpha \end{aligned} \tag{5}$$

Using the aforesaid property, (1) can be expressed as,

$$y = AS^T\alpha \tag{6}$$

Since the solution is sparse, it can be recovered using l_1-norm minimization (7). Once the sparse transform coefficient vector is obtained, the signal can be reconstructed from α using the synthesis equations

$$\min_\alpha ||\alpha||_1 \quad \text{subject to} \quad y = AS^T\alpha \tag{7}$$

This is called the synthesis prior formulation. It is well understood theoretically by CS researchers and is popular in many applied research areas as well.

Unfortunately this synthesis prior formulation is too rigid. Not all transforms are orthogonal/tight-frame and hence the signals cannot be expressed as analysis–synthesis pairs. For example medical images are assumed to be piece-wise smooth, and hence have very sparse gradients. Gradient operators cannot be expressed in analysis–synthesis relationship. There are other linear sparsifying transforms like Gabor transform, cubic-splines, contourlets etc. which are not orthogonal or tight-frame. In such cases, the analysis equation holds, but the synthesis equation does not; therefore even though $\alpha = Sx$ is sparse, it is not possible to synthesize $x = S^T\alpha$. Therefore, for such transforms it is not possible to frame a synthesis prior problem such as (7).

The alternate analysis prior problem is more generalized and can be formulated for any linear sparsifying transform (need not be orthogonal or tight-frame). The analysis prior formulation is as follows:

$$\min_x ||Sx||_1 \quad \text{subject to} \quad y = Ax \tag{8}$$

Here S is the sparsifying transform and Sx is sparse.

It must be noted that the analysis and the synthesis prior formulations are exactly equivalent for orthogonal transforms but they are different for tight-frame transforms. In practice it has been found that analysis prior yields better results [11,12].

In spite of these advantages analysis prior is not popular in CS. It is only in recent times that the analysis prior problem is being noticed by the signal processing community [13,14]. Theoretical understanding in this topic is still not matured and there are only a few applied areas where this formulation is being applied. We will not get into the theoretical intricacies of analysis prior optimization. In layman's terms, the analysis prior solves for the signal itself whereas the synthesis prior solves for the sparse transform coefficients of the signal.

3. Literature review

One of the earliest works that applied CS on EEG signals is [1]. The focus of that paper was on EEG compression. They projected the EEG signal onto an i.i.d Gaussian basis (to a lower dimension for compression) and used CS to recover the EEG signal by exploiting the signal's sparsity in the Gabor domain. This paper also mentioned about the possibility of jointly reconstructing the EEG signals by utilizing the inter-channel correlation. They assumed a dictionary learning based approach for this purpose; nothing concrete regarding the joint recovery was discussed in this paper.

The possibility of using CS for wireless tele-monitoring of EEG signals has been discussed in [2,3]. In these studies it was shown, how inter-channel correlation can be exploited to improve reconstruction accuracy. The recovery was posed as a synthesis prior problem. The limitation of synthesis prior has been discussed before. Owing to these constraints, there was a limited choice on the sparsifying transforms. In [2] DCT was used and in [3] wavelets was used as a sparsifying transform for EEG.

In [4], different sprasifying transforms were compared as sparsifying basis for EEG – wavelets, Gabor, splines; it was reported that Gabor yielded the best reconstruction results. This paper (and others) formulated EEG reconstruction as a synthesis prior problem.

Till now, we have discussed studies where the sampling and compression were two different stages of the information processing pipeline. The EEG signal was sampled normally, later it was projected onto an i.i.d Gaussian matrix for compression. In [5], it was shown the compression and the sampling process can be done in one go using Slepian basis. True to the spirit of CS, instead of collecting 'EEG samples' they suggested a framework where EEG signal could be 'measured' directly in the Slepian basis. They too posed the reconstruction as a synthesis prior problem and hence were restricted to the usage of wavelet transforms as the sparsifying basis.

A recent study [6] addresses the same problem as ours. The objective was to reconstruct the EEG signals remotely from their sub-sampled projections. It uses Gabor transform as the sparsifying basis. This was studied earlier; the only novelty of [4] is that, they proposed a computationally cheap method to perform Independent Component Analysis denoising prior to compression and subsequent transmission.

There are two problems with the aforesaid works. The first problem is technical. Gabor dictionary is not an orthogonal/tight-frame transform; therefore it is erroneous to pose EEG reconstruction on Gabor basis as a synthesis prior problem. But the aforesaid studies [1,4,6] do that.

The second problem lies deeper. The other studies [2,3,5] use DCT or wavelets as the sparsifying transform, hence they are justified in posing the reconstruction as a synthesis prior problem. However these studies are fraught with a deeper issue. A large of EEG signal analysis problems are based on the Gabor transform

– Gabor coefficients (of EEG signals) are used for seizure detection [15,16], emotion analysis [17], and for other tasks [18,19]. Following these works, it is reasonable to assume that the EEG signal after reconstruction would be subjected to Gabor transform for further analysis. Needless to say, the reconstruction should be such that the Gabor coefficients of the reconstructed be as close to Gabor coefficients of the original signal as possible. When DCT or wavelets are used as the sparsifying basis, this criterion is not met. CS reconstruction can only guarantee preservation of the high valued transform coefficients of the sparsifying basis; i.e. if one uses DCT, wavelets or any other sparsifying basis for EEG signal reconstruction, there is no guarantee that the high valued of the Gabor coefficients will be preserved between the original and the reconstructed signals. Thus even though [2,3,5] are technically correct reconstruction techniques, they are fraught with this flaw.

In a slightly different note, there is a paper which addresses a similar problem. The EEG signals are collected remotely and the data analysis is performed centrally [7]. What is interesting about this study is that it was shown that certain data analysis, e.g. seizure detection, can be directly performed in the compressed domain; it is not required to reconstruct the signal. This is a smart study, but with limited scope. For general purpose applications, the EEG signal needs to be reconstructed.

4. Proposed solution

We mentioned before that in a WBAN the energy is expended in sampling, processing and transmission. In this work, we do not modify the sampling process; we are interested in reducing the energy consumption in processing and transmission.

For energy efficient transmission we need to compress the signal. For reasons mentioned before, we look into CS based compression techniques. Following CS theory, the sampled EEG signals need to be ideally projected onto a lower dimension by an i.i.d Gaussian matrix. However storing and operating on a dense Gaussian matrix is impractical. In [20] an efficient CS measurement matrix was proposed – it consists of '*d*' ones at random positions in each rows of the matrix. There is no theoretical proof regarding the optimality of such a sparse binary matrix as a CS measurement operator, but it was found that it yields very good results (as good as an i.i.d Gaussian matrix). The benefit of using such a sparse matrix over a dense one is its computational and storage gains – making it an ideal choice for our problem. In [2,6] it was found that such a matrix gives good results for EEG reconstruction.

For reconstruction one needs to choose an appropriate sparsifying transform. We have argued earlier why DCT and wavelets is not a good choice for sparsifying transform. Since EEG signal analysis is almost always performed in the Gabor domain, using the Gabor basis as the sparsifying transform is more sensible, since this would guarantee that the high valued Gabor coefficients (required for analysis) are preserved. Unfortunately previous studies using Gabor transform as the sparsifying basis erroneously posed the reconstruction as a synthesis prior problem. In this work, we propose to pose EEG reconstruction as an analysis prior problem

$$\min_x ||Gx||_1 \quad \text{subject to} \quad y = Bx \tag{9}$$

Here is G is the Gabor basis and B is the sparse binary measurement matrix. There are not many algorithms to solve the analysis prior problem. In [11] a Chambolle-type algorithm was developed to solve the unconstrained analysis prior problem. Later, we modified it to solve the constrained analysis prior problem [12]. The same algorithm is used here to solve the analysis prior problem (9). Even without the requirement of further processing via Gabor transform, it has been reported previously [11,12] that analysis prior formulation yields better reconstruction accuracy compared to the synthesis prior.

Table 1
Comparative reconstruction results (NMSE).

Method	Compression ratio	
	2:1 (mean, std)	4:1 (mean, std)
Piecemeal reconstruction [2]	0.314, ±0.122	0.522, ±0.162
Joint reconstruction [2]	0.112, ±0.060	0.288, ±0.148
Proposed analysis prior	0.160, ±0.024	0.318, ±0.086

In almost all the papers [1–6] the signal reconstruction is measured in terms of Normalised Mean Squared Error (NMSE) or some of its variants. It must be remembered that signal reconstruction is only an intermediate step in the signal processing pipeline. The reconstructed signal is analyzed later on. Measuring the quality of the signal reconstruction alone does not make any sense. A fairer comparison would be see how the different reconstruction techniques perform when the reconstructed signals are subjected to specific analysis problems. In this paper, we deviate from the evaluation methodology previously used. We will report NMSE values, but we will also report the results from a standard EEG classification problem to evaluate accuracy.

5. Experimental evaluation

Our aim is to test if our proposed analysis prior formulation for EEG reconstruction is better than previous synthesis prior based techniques. All previous studies reported their results based on NMSE. We argued that this is not the best metric. If there was no WBAN involved, the EEG signal would be acquired and analyzed; may be for detecting epileptic seizures or understanding mental tasks. In this scenario, we have an intermediate WBAN which acquires and transmits the signal. For further analysis we need to ensure that the reconstructed (after transmission) signal is almost a replica of the original. However, reconstruction is an intermediate step and a good NMSE does not necessarily guarantee a good result in the final EEG analysis task. In this work, we report NMSE's since it has been a standard metric to evaluate reconstruction. However, we also go a step further and see how the task of EEG classification varies between the original and the reconstructed signals. We use a simple SVM based classification method for the purpose [21].

The experiments are carried out on the BCI III dataset 1; it was basically a binary classification problem (for more details see [22]). The classification method [21] was a participant at the BCI III competition.

We compared our analysis prior formulation with the method proposed in [2]. There in two reconstruction algorithms are proposed – in the first one the EEG signals are reconstructed piecemeal, in the second one the channel correlations are exploited to jointly reconstruct all the EEG signals. It was shown that the joint reconstruction yields better results. In this work we compare our work with both separate and joint reconstruction [2]. In [2] Discrete Cosine Transform (DCT) is used as the sparsifying basis; reconstruction is formulated as a synthesis prior problem.

We remind the reader that we use Gabor as the sparsifying basis. The parameters for the Gabor transform were fixed empirically; we found that the best results are obtained at the highest resolution without any frequency or shift redundancy.

The experimental results are showed in Table 1. Normalized Mean Squared Error (NMSE) is used as the metric for evaluating reconstruction accuracy. For each configuration, the sparse binary matrix was generated 1000 times, and the mean and standard deviations (std) are reported. The results are shown for two under-sampling (compression) ratios – 2:1 (50% under-sampling) and 4:1 (25% under-sampling).

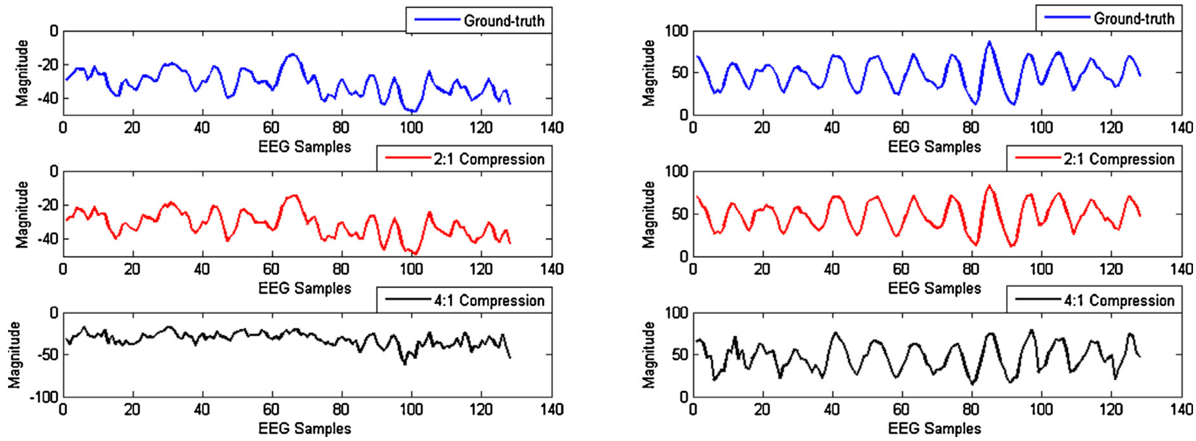

Fig. 1. EEG reconstruction results. Top – ground-truth, Middle – 2:1 compression, Bottom – 4:1 compression.

Our proposed method is a piecemeal reconstruction approach, i.e. we do not exploit inter-channel correlation while reconstruction. When comparing piecemeal reconstruction, our method is considerably better than [2] (see rows 1 and 3). In joint reconstruction, more information (inter-channel correlation) is incorporated in the reconstruction process. Thus the reconstruction results are likely to be better. This can be seen from the table (row 2). Since we do piecemeal reconstruction, we are slightly worse (in reconstruction accuracy) compared to the correlation based formulation of [2].

However, as mentioned before, in this scenario, NMSE is not the best measure for accuracy. We carried out classification using algorithm [21] on the BCI III dataset 1. The classification was carried out on the groundtruth as well as on the reconstructed EEG data. The classification results are shown Table 2.

The classification algorithm when applied on the groundtruth dataset yields a reconstruction accuracy of 81% (the same result is reported in the BCI III competition website [22]). When the same algorithm is applied on the reconstructed data, our method yields the best accuracy. It is better than both the piecemeal and the joint reconstruction techniques proposed in [2]. The classification accuracy at 2:1 compression ratio is worth noting. At a higher compression ratio (4:1), the classification results are worse and is slightly better than random guessing.

This phenomenon can be better understood by looking at examples of actual reconstructions. In Fig. 1, we shows the groundtruth and reconstructed EEG signal for two compression ratios. It can be seen that for 2:1 compression ratio, the original structure (groundtruth) is almost completely preserved. But for higher compression ratio 4:1 the structure is lost. Thus it is understandable why we have good classification at the lower compression ratio (almost as good as groundtruth), but poor accuracy at 4:1 compression ratio – since the structure is completely lost, the performance borders that of random guessing.

Table 2
Classification accuracy in %.

Method	Classification accuracy	
	Compression 2:1	Compression 4:1
Groundtruth	81% (no compression)	
Piecemeal reconstruction [2]	69%	50%
Joint reconstruction [2]	76%	54%
Proposed analysis prior	79%	54%

6. Conclusion

Here we show how the reconstruction of EEG signals can be improved by posing it as an analysis prior problem as opposed to prevalent synthesis prior techniques [1–6]. Previous studies reported results based on reconstruction accuracy. Here we argue that reconstruction is only an intermediate step; the final goal is EEG signal analysis. Therefore instead of only reporting results on reconstruction accuracy, one should evaluate how the performance of signal analysis changes.

Our proposed technique reconstructs each EEG signal separately; it does not account for inter-channel correlation. We compare our results with [2]; there in it is showed that joint reconstruction of the EEG signals which exploits inter-channel correlation yields better results. Our method is only slightly worse than theirs [2] in terms of reconstruction accuracy. This is understandable; they [2] uses more information (inter-channel correlation) during reconstruction and thereby achieves better results. However, when we compare our proposed technique with [2] in terms of classification accuracy (signal analysis) we see that our method is better.

There is scope for further research in this area. It has been shown in [2,3] that better reconstruction is achieved when inter-channel correlation is exploited during reconstruction. In this work, we do not use this information, we reconstruct each channel separately. However, it is easy to extend this work and formulate an analysis prior joint reconstruction problem. This would further improve the reconstruction accuracy; it is likely that it would also improve the classification accuracy of EEG signals.

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