Elastic Net Formulation for MRI Reconstruction

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INTRODUCTION

Compressed Sensing (CS) based techniques exploit the sparsity of the underlying MR image in the wavelet domain in order to reconstruct it from partial K-space data. This is perfect for recovering arbitrarily sparse signals having no sparsity structure. However, high-valued wavelet coefficients of MR images have a tree-structure, i.e. if the wavelet coefficient at higher scale is of high value, the corresponding coefficient at lower scales will also be of high valued. Thus the sparse wavelet coefficients of MR images are hierarchically correlated. Standard l_p-minimization is too generalized to exploit this correlation.

There have been prior studies on recovering such tree-sparse wavelet coefficients [1]. However finding exact tree-structured sparse signals is computationally challenging and is not scalable for practical MRI reconstruction. To bypass this tree-search requirement, a more recent work [2], proposed reconstructing the hierarchically correlated wavelet coefficients by dividing them into overlapping groups. In this work, we do not impose such strong group-sparsity constraints. We propose an elastic-net formulation which encourages grouping but does not enforce it.

THEORY

The elastic-net formulation was proposed in [3] to overcome the shortcomings of l_1-minimization (LASSO). In general l_p-minimization cannot recover a group of correlated variables; implying that for MRI reconstruction it may not recover all the hierarchically correlated high valued wavelet coefficients. To overcome this shortcoming elastic-net proposes the following formulation: $\min_{\alpha} \|\alpha\|_p^p + \eta \|\alpha\|_2^2$ s.t. $\|y - FW^T\alpha\|_2^2 \le \varepsilon$, where y is the K-space data, F is the partial Fourier operator, W the wavelet transform, α is the vector of sparse coefficients and ε is the noise parameter. In this formulation, the l_p-norm enforces sparsity on α and the l_2-norm encourages a grouping effect. This grouping effect allows recovery of all the hierarchically correlated wavelet coefficients. It must be remembered that the original elastic-net formulation uses the convex l_1-norm; the l_p-norm is our generalization.

It has been observed that the instead of employing such a synthesis prior formulation, better reconstruction results can be achieved by the analysis prior formulation [4]. Thus we propose an analysis prior formulation elastic-net: $\min_{x} \|Wx\|_p^p + \eta \|Wx\|_2^2$ s.t. $\|y - RFx\|_2^2 \le \varepsilon$

The analysis and the synthesis prior are theoretically equivalent for orthogonal wavelets but they are different for tight-frames.

METHOD

In this work, we have performed an experimental study on three different MR images - rat, brain and phantom. The ground-truth data is collected by fully sampling the k-space on a uniform Cartesian grid. The size of all the images are 256 x 256; for all the images spin echo sequence was used for acquisition. The T2 weighted rat's spinal cord was acquired by a 7T scanner at UBC's MRI Lab with echo time of 13ms. The brain image and the phantom have been used in [5]. For the experiments the variable density random sampling with 3 fold acceleration factor is used for simulating partial sampling of the K-space.

RESULTS

Our method requires specification of three parameters - p and η. It was found that p = 0.8 yields the best results. The value of η = 0.25 was found via the L-curve method. We use the complex dual-tree wavelet as the sparsifying transform. The results of quantitative evaluation are given in Table 1. Normalized Mean Squared Error (NMSE) is used for evaluating MRI reconstruction. We compared our approach with the overlapping group-sparse recovery method [2].

The overlapping group-sparsity prior [2] enforces the most stringent constraints on the reconstruction problem and yields very good results. Our elastic-net formulation does not enforce strict group-sparsity but encourages grouping effect. Our synthesis prior formulation is only marginally worse than [2]; but the analysis prior formulation yields considerably superior results than [2].

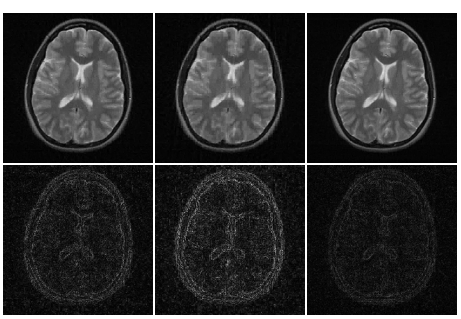

Table 1. Reconstruction Error

Dataset	Group-sparse [2]	Synthesis prior elastic-net	Analysis prior elastic-net
Spine	0.08	0.09	0.05
Brain	0.13	0.15	0.09
Phantom	0.15	0.18	0.11

For qualitative evaluation, the reconstructed and the difference images are shown in the adjacent figure. Owing to limitations in space, we only show the results for the brain image. The qualitative observations corroborate the quantitative results.

CONCLUSIONS

In this work, we propose to reconstruct MR images via the elastic-net. We generalize the elastic net to incorporate non-convex sparsity priors and also propose an alternate analysis prior formulation. The elastic-net formulation is capable of recovering hierarchically correlated wavelet coefficients and thereby results in superior reconstruction.

REFERENCES

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