Split Bregman Algorithms for Sparse / Joint-sparse and Low-rank Signal Recovery: Application in Compressive Hyperspectral Imaging

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ABSTRACT

In this work we derive algorithms for solving two problems - the first one is the combined l_1-norm (sparsity) and nuclear norm (low rank) regularized least squares problem and the second one is the $l_{2,1}$-norm (joint sparsity) and nuclear norm regularized least squares problem. There are no efficient general purpose solvers for these problems; our work plugs this gap by deriving Split Bregman based algorithms for solving the said problems. Both algorithms are applicable for recovering hyperspectral images from their compressive measurements obtained via the single pixel camera. We show that our proposed techniques significantly outperform previous methods in terms of recovery accuracy.

***Index Terms*—** Low rank matrix recovery, Sparse Recovery, Joint Sparse Recovery, Hyperspectral Imaging

1. INTRODUCTION

In the past half a decade there has been enormous interest in theory, algorithms and applications of linear inverse problems where the solution is sparse,

$$y_{n\times 1} = A_{n\times N} x_{N\times 1} + \eta_{n\times 1},\ n < N \quad (1)$$

Here one is interested in solving x, given the measurements y and the linear system A; the measurement is assumed to be corrupted by white Gaussian noise $\eta \sim \quad)$.

In Compressed Sensing (CS) [1, 2] it is assumed that the solution x is sparse. And theoretical studies in this area prove that given some constraints on A, it is possible to recover the sparse solution by solving the following l_1-norm minimization problem,

$$\min_x \|x\|_1 \text{ subject to } \|y - Ax\|_2^2 \le \varepsilon,\ \varepsilon = n\sigma^2 \quad (2)$$

where $\|x\|_1 = \sum_{i=1}^{N} |x(i)|$.

The problem of low-rank matrix recovery is closely related to CS. Here it is assumed that the solution x is of low rank (arranged in a suitable matrix form); i.e. it is assumed that a vector formed by the singular values of x is sparse. Theoretical studies [3, 4] show that the low-rank solution can be recovered by nuclear norm minimization,

$$\min_X \|X\|_* \text{ subject to } \|y - Ax\|_2^2 \le \varepsilon,\ x_{N\times 1} = vec(X_{p\times q}) \quad (3)$$

where $\|X\|_* = \sum_{i=1}^{\min(p,q)} |s(i)|,\ X = USV^T$ and $s = diag(S)$.

Sparse recovery and low-rank matrix recovery finds applications in a wide variety of topics in electrical engineering and computer science. Deriving efficient algorithms for solving sparse (2) and low-rank (3) signal recovery has matured over the years, but still is considered to be an active area of research.

Recent studies go a step further and model signals as a superposition of sparse and low-rank components, i.e.

$$X = S(sparse) + L(low - rank) \quad (4)$$

Here the problem is to separate these two components; the problem is popularly called the Robust Principal Component Analysis (RPCA) [5, 6]. The optimization problem to be solved is as follows:

$$\min_{L,S} \|L\|_1 + \lambda \|S\|_* \text{ subject to } X = L + S \quad (5)$$

A more difficult problem is to separate these components when the measurement is a lower dimensional projection [7]. There are several fast and efficient algorithms to solve the RPCA problem - these algorithms are termed 'principal component pursuit' [8, 9].

In this work, we are interested in signal models which are not superpositions of sparse and low-rank components; we look at signals which are simultaneously sparse AND low-rank; i.e. we are interested in solving (1) but we assume that x is simultaneously sparse and low-rank. Such a problem arises in dynamic Magnetic Resonance Imaging (MRI) [10-13]. For recovering a sparse and low-rank solution, one usually needs to solve an optimization problem of the following form:

$$\min_X \|y - Ax\|_2^2 + \lambda_1 \|x\|_1 + \lambda_2 \|X\|_* \quad (6)$$

Here the l_1-norm promotes a sparse solution whereas the nuclear norm encourages a low-rank solution. The user assigns the relative importance of the two penalties by fixing λ_1 and λ_2.

In hyper-spectral imaging (HSI) [14] the problem is different. Here the signal is joint-sparse as well as is low-rank. The optimization problem to be solved is [14-16]:

$$\min_{X} \|y - Ax\|_2^2 + \lambda_1 \|X\|_{2,1} + \lambda_2 \|X\|_* \quad (7)$$

where x=vec(X), $\|X\|_{2,1} = \sum_{j=1}^{N} \|X^{j\rightarrow}\|_2$ and j→ denotes the j[th] row row.

This is a multiple measurement vector (MMV) recovery problem where X has a common sparse support, i.e. X is row-sparse; furthermore X is low-rank as well. The mixed $l_{2,1}$-norm for joint/row sparse penalty is well known [14, 15].

Even though sparse / joint sparse and low-rank signal recovery is applicable to some major problems in scientific imaging there are no off-the-shelf efficient algorithms to solve them. The papers in dynamic MRI only solve the problem (6) approximately. In [10] the algorithm is tailored for Total Variation minimization (sparsity penalty); in [11] the low-rank penalty is approximated by matrix factorization which leads to a highly non-convex formulation with no convergence guarantees; in [12, 13] the solution is derived based on iterative re-weighted least squares techniques which can only promise asymptotic convergence. The papers on joint sparse and low-rank recovery [14] are theoretical in nature, their recovery algorithm, is not very efficient.

Given this current scenario, there is a need to derive efficient algorithms to solve sparse / joint sparse and low-rank recovery problems. This is the major contribution of this work and will be described in section 2. As a practical example, we will apply these techniques to the HSI problem in section 3. The experimental results will be examined in section 4. The conclusions of the work will be discussed in section 5.

2. SPLIT BREGMAN ALGORITHMS

We want to solve the sparse and low-rank recovery problem (6) and joint sparse and low-rank recovery problem (7). We repeat the problems for the sake of convenience.

$$\min_{X} \|y - Ax\|_2^2 + \lambda_1 \|x\|_1 + \lambda_2 \|X\|_*$$

$$\min_{X} \|y - Ax\|_2^2 + \lambda_1 \|X\|_{2,1} + \lambda_2 \|X\|_*$$

In practical scenarios, the signal is hardly ever sparse in itself; most often it is sparse in a transform domain. If the transform is orthogonal (e.g. Fourier, Wavelet etc.) it is possible to express the recovery as a synthesis prior problem such as (6) and (7). But for general purpose (non-orthogonal) linear operators like Gabor or Total Variation penalty, it is not possible to frame a synthesis prior problem. In such cases, one needs to formulate them as analysis prior problems [17, 18],

$$\min_{X} \|y - Ax\|_2^2 + \lambda_1 \|Dx\|_1 + \lambda_2 \|X\|_* \quad (8)$$

$$\min_{X} \|y - Ax\|_2^2 + \lambda_1 \|DX\|_{2,1} + \lambda_2 \|X\|_* \quad (9)$$

Here D is the dictionary / linear operator where the signal is assumed to be sparse. It is easy to observe that the synthesis prior is just a special case of the analysis prior formulation where D is identity. Therefore, in this work we will derive algorithms to solve the general purpose analysis prior formulations (8) and (9).

We follow the Split Bregman approach to solve the optimization problems. This approach has not been used before for solving these problems. Owing to limitations in space, we do not go into the background of Split Bregman. First we derive an algorithm to solve (8). The derivation for (9) will be similar and will be done later.

We solve (8) by Bregman type variable splitting with Alternating Directions Method of Multipliers (ADMM) [19]. We introduce two proxy variables - *p=vec(P)* and *q=vec(Q)* for the two penalty functions respectively. We add terms relaxing the equality constraints of each quantity and its proxy, and in order to enforce equality at convergence, we introduce Bregman relaxation variables B_1 and B_2. The new objective function is:

$$\begin{aligned}\min_{X,P,Q} & \|y - Ax\|_2^2 + \lambda_1 \|Dp\|_1 + \lambda_2 \|Q\|_* + \\ & \gamma_1 \|P - X - B_1\|_F^2 + \gamma_2 \|Q - X - B_2\|_F^2\end{aligned} \quad (10)$$

This allows the problem (10) to be split into an alternating minimization of the following (easier) subproblems:

$$\min_{X} \|y - Ax\|_2^2 + \gamma_1 \|P - X - B_1\|_F^2 + \gamma_2 \|Q - X - B_2\|_F^2 \quad (11)$$

$$\min_{P} \lambda_1 \|Dp\|_1 + \gamma_1 \|P - X - B_1\|_F^2 \quad (12)$$

$$\min_{Q} \lambda_2 \|Q\|_* + \gamma_2 \|Q - X - B_2\|_F^2 \quad (13)$$

The subproblem (11) is easy to solve; it is just a least squares problem that can be solved efficiently using any conjugate gradient algorithm. The subproblem (12) is an analysis prior denoising problem. The technique to solve this is borrowed from [20]:

$$z = (\frac{\gamma_1}{\lambda_1} \Lambda^{-1} + cI)^{-1} (cz + D(vec(X + B_1) - D^T z))$$

$$p \leftarrow vec(X + B_1) - D^T z$$

where $\Lambda = diag(|\, D\, vec(X + B_1)\,|^{-1})$ and c is the maximum eigenvalue of $D^T D$.

The subproblem (13) is a nuclear norm minimization. The algorithm to solve this was derived in [21]. The method is called singular value shrinkage.

$$USV^T = X + B_2$$

$$\Sigma \leftarrow Soft(S, \lambda_2 / \gamma_2)$$

$$Q = U \Sigma V^T$$

Soft-thresholding is applied on the singular values of the matrix X+B_2; Q is updated by recomposing the matrix using the singular vectors and the thresholded singular values.

This concludes the derivation for solving (8). Solving (9) is very similar. Applying the Split Bregman technique to (9) leads to the following:

$$\min_{X,P,Q} \|y-Ax\|_2^2 + \lambda_1 \|Dp\|_{2,1} + \lambda_2 \|Q\|_* + \gamma_1 \|P-X-B_1\|_F^2 + \gamma_2 \|Q-X-B_2\|_F^2 \quad (14)$$

This too can be segregated into 3 subproblems. The first and the third subproblems remain exactly the same as before, i.e. same as (11) and (13). The only change is in the second subproblem that solves the joint sparsity penalty:

$$\min_{P} \|y-Ax\|_2^2 + \lambda_1 \|Dp\|_{2,1} + \gamma_1 \|P-X-B_1\|_F^2 \quad (15)$$

The solution to (15) has already been derived in [18].

$$Z = (\frac{\gamma_1}{\lambda_1}\Lambda^{-1} + cI)^{-1}(cZ + D(X+B_1-D^T Z))$$

$$X \leftarrow X + B_1 - D^T Z$$

Here $\Lambda = diag(\|D(X+B_2)^{j\rightarrow}\|_2^{-1})$, i.e. the diagonal elements contain the l_2-norms of the rows in $D(X+B_2)$.

The final step for both problems is to update the Bregman variables:

$$B_1 \leftarrow X + B_1 - P \quad (16)$$

$$B_2 \leftarrow X + B_2 - Q \quad (17)$$

This concludes our derivation of both the algorithms. We have used upper-case symbols (X and P) for matrices and lower-case symbols for corresponding vectors (x and p).

3. HYPER-SPECTRAL IMAGING

In this work we address the problem of Compressive Hyperspectral Imaging (HSI). A modification of the single pixel camera [22] is used to capture images in different spectral bands. Formally, this is expressed as:

$$y_c = B_c x_c + \eta_c,\ \text{c=1...C} \quad (18)$$

where c denotes the c^{th} spectral band, x_c is the corresponding image, B_c is the corresponding binary projection operator realized by the single pixel camera [22], y_c is the obtained measurement and η_c is the noise.

This can be succinctly represented in MMV form:

$$Y = BX + \eta \quad (19)$$

where Y is a matrix formed by stacking the y_c's as columns, similarly X is formed by stacking the x_c's as columns and B is a block diagonal matrix with B_c's along the diagonals.

Each of the spectral band images are sparse in the wavelet domain. Therefore incorporating the wavelet transform into (19), one can write

$$Y = BW^T\alpha + \eta \quad (20)$$

where W is the wavelet transform and α is the sparse wavelet transform coefficients (still in arranged in MMV form).

In [14] it is argued that since, the spectral bands are similar to each other, all the images will have the sparsity signature in the transform domain. This leads to a signal model where the matrix α is joint / row sparse. Such a model had been proposed earlier for color imaging [23, 24] and the extension to HSI is trivial. The novelty of [14] lies in the fact that, they recognized that since the HSI datacube is spectrally correlated, the signal X is low-rank; spectral correlation leads to linear dependent columns in X. In order to exploit both joint sparsity and low-rank information, [14] proposed to recover α via:

$$\min_{\alpha} \|Y - AW^T\alpha\|_2^2 + \lambda_1 \|\alpha\|_{2,1} + \lambda_2 \|Mat(\alpha)\|_* \quad (21)$$

'Mat' denotes the vector has been converted to matrix form.
This is the synthesis prior form; same as (7). In this work, we propose to solve the analysis prior formulation instead:

$$\min_{X} \|Y-AX\|_2^2 + \lambda_1 \|WX\|_{2,1} + \lambda_2 \|X\|_* \quad (22)$$

In [25] a Kronecker Compressed Sensing (KCS) formulation was proposed to solve the same problem. Since the different spectral bands are correlated, the variation along the spectral direction is smooth. This smooth variation can be compactly captured by very few Fourier coefficients. Also, each spectral band is sparse in wavelet domain. Combining these two ideas, [25] proposed to maximally decorrelate the hyperspectral datacube by applying wavelet transform along the columns of X and applying Fourier transform along the rows of X. Mathematically this leads to the Kronecker product formulation:

$$\beta = WXF = F^T \otimes W\ vec(X)$$

$$\Rightarrow vec(X) = \left(F^T \otimes W\right)^T \beta$$

Here β is the sparse coefficient vector after applying the different sparsifying transforms in different directions. In [25], the recovery was posed as synthesis prior problem:

$$\min_{x} \|\beta\|_1 \text{ subject to } \left\|vec(Y) - B\left(F^T \otimes W\right)^T \beta\right\|_2^2 \le \varepsilon \quad (23)$$

In this work, we propose to enhance the performance of the KCS technique by imposing an additional penalty on low-rank. Moreover, we propose to solve the analysis prior formulation. Our recovery is posed as:

$$\min_{X} \|Y-AX\|_2^2 + \lambda_1 \left\|\left(F^T \otimes W\right) vec(X)\right\|_{2,1} + \lambda_2 \|X\|_* \quad (24)$$

3. EXPERIMENTAL RESULTS

3.1 Synthetic Data

First we show results for the problem where the signal model is assumed to both sparse and low-rank; it is shown that combining sparsity with low-rank property yields better results than employing sparsity or rank-deficiency alone. The size of the matrix is fixed at 50 X 50; number of non-zero elements is fixed at 10% and the ranks are varied. A binary measurement operator is employed for projecting the matrix onto a lower dimension. The sampling ratio is 50%. The comparative reconstruction accuracies are shown in Table 1. For each of the problems, the ground truth data is generated 100 times, and the mean errors are reported. Error is measured in terms of normalized mean squares error (NMSE). The parameters used for our algorithm are γ_1= γ_2=1e-4, λ_1=1e-4 and λ_2=10. The l_1-norm is minimized by SPGL1 [26], and the nuclear norm by [27].

Table 1. NMSE from different reconstruction techniques

Rank	Nuclear norm	l_1-norm	l_1-norm + nuclear norm (proposed)
2	5.67×10^{-5}	2.76×10^{-1}	6.33×10^{-6}
5	1.08×10^{-4}	2.95×10^{-1}	1.19×10^{-5}

We show similar experimental results for the joint sparse and low-rank recovery algorithm; the results from our proposed method is better than recovery with only nuclear norm minimization (low rank penalty) or only joint sparse recovery (solved using SPGL1 [26]). Here the size of the matrix is 50 X 50, with 10% of the rows to be non-zeroes; the rank is varied. The data is generated 100 times and the average errors are reported in Table 2. The parameters used in our algorithm are $\gamma_1=\gamma_2=10$ and $\lambda_1=\lambda_2=1e-4$.

Table 2. NMSE from different reconstruction techniques

Rank	Nuclear norm	$L_{2,1}$-norm	l_1-norm + nuclear norm (proposed)
2	3.96×10^{-5}	1.19×10^{-1}	5.6×10^{-6}
5	1.97×10^{-2}	1.19×10^{-1}	5.3×10^{-3}

Tables 1 and 2, show that recovery via sparse / joint-sparse penalties yields considerably poor results and do not compare with the others. Our proposed method yields the best results, which is about an order of magnitude better than nuclear norm minimization (only low-rank penalty).

3.2 Compressive Hyper-spectral Imaging

We used three hyperspectral images for experimental evaluation. Two images are from the AVIRIS (Airborne Visible/Infrared Imaging Spectroscope) sensor and one is from the HYDICE (Hyperspectral Digital Imagery Collection Experiment) sensor. All three images are freely available from [28][29]. Both the AVIRIS and the HYDICE sensors capture wavelengths ranging from 400 nm to 2500 nm. The first AVIRIS image is of Moffett Field, CA and has 224 continuous bands. The second AVIRIS image is of Indian Pine Test site and has 220 continuous bands. The third image was of Washington DC Mall taken from HYDICE sensor. This image has 191 bands (after rejecting noisy bands).

The experimental results are shown in Tables 3 and 4. The recovery accuracy is measured in terms of PSNR. For each of these images a small continuous portion of 160 X 160 pixels was chosen for the experiments. Measurements from the single pixel camera was simulated using Binary projection matrices.

The images from the binary projections were recovered using different techniques. The baseline method is the Kronecker CS (KCS) [25]. As mentioned before, it was shown in [14] that a low-rank + joint-sparse ($l_{2,1}$+*NN*) recovery technique yields good results for compressive hyper-spectral imaging. In this paper we show how the low-rank + Kronecker CS (l_1+*NN*) model can be used for recovering hyperspectral images as well. In [14] a proximal projected gradient algorithm (PPXA) is proposed to solve the required $l_{2,1}$+*NN* minimization problem. In this work, we solve both the $l_{2,1}$+*NN* and the l_1+*NN* minimization problems using Split Bregman technique. The parameters used for our algorithm are $\gamma_1=\gamma_2=10$ and $\lambda_1=\lambda_2=1e+4$.

Table 3. PSNR for 25% sampling

Image	l_1+*NN* (Proposed)	$l_{2,1}$+*NN* (Proposed)	l_1 (KCS)	$l_{2,1}$+*NN* (PPXA)[14]
Moffett	31.61	31.6	29.21	30.0
Indian Pine	35.92	36.0	31.66	33.98
Washington	36.84	37.18	23.36	30.28

Table 4. PSNR for 50% sampling

Image	l_1+*NN* (Proposed)	$l_{2,1}$+*NN* (Proposed)	l_1 (KCS)	$l_{2,1}$+*NN* (PPXA)[14]
F08small	36.30	36.16	30.11	33.9
92AV3C	40.09	40.25	31.79	37.18
Washington	41.89	42.11	26.12	38.59

The results show that KCS [25] yields the poorest recovery. The $l_{2,1}$+*NN* recovery solved using proximal projected gradient algorithm [14] improves over KCS but does not match the recovery results from our proposed Split Bregman algorithms. For visual clarity we show the reconstructed results for Washington DC, Mall with 25% sampling for three bands shown in pseudo colours. We have omitted the result from [14] ($l_{2,1}$+*NN* using PPXA), because our algorithm yields better results. It can be seen that the KCS technique cannot recover the original image very well while our proposed methods can.

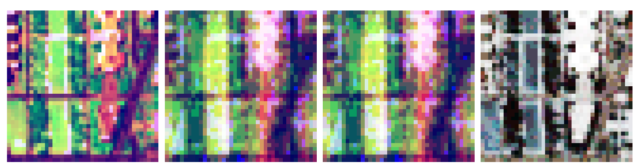

Fig. 1 Left to Right: Original, $l_{2,1}$+*NN*, l_1+*NN* and KCS.

4. CONCLUSION

The main contribution of this work is to derive efficient algorithms to solve under-determined system of linear equations where the solution is known to sparse / joint-sparse and low-rank. It is to be noted that this differs from the RPCA signal model where the solution is assumed to be a super-position of sparse and low-rank components.

Sparse / joint-sparse and low-rank signal models arise in several areas of scientific imaging [10-15]. Unfortunately, there are no efficient algorithms to solve the generic problems. In this work, we derive efficient algorithms to solve these problems based on the Split Bregman technique. The Matlab implementation of these algorithms is available at [30, 31].

Compressive Hyperspectral imaging can be modeled either as a sparse and low-rank recovery problem or as a joint-sparse and low-rank recovery problem. We apply our proposed algorithms to solve the said problem. We find that our method yields significantly better results than previous techniques. In imaging problems an improvement in 0.5 dB to 1 dB is considered good; here we improve around 2dB on average

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