Generalized Synthesis and Analysis Prior Algorithms with Application to Impulse Denoising

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Abstract—This work proposes generalized synthesis and analysis prior algorithms using the split-Bregman technique for applications in impulse noise reduction. Impulse denoising is formulated as minimizing a Lp-regularized Lq-norm data mismatch term. The Lq-norm mismatch arises owing to the fact that the noise is sparse. The Lp-norm exploits the prior information that the image is sparse in a transform domain. The proposed methods have been used to reduce salt and pepper noise as well as random valued impulse noise. Peak signal to noise ratio and structural similarity index have been used to quantitatively evaluate the recovery results. A comparative study with existing IRN algorithm suggests the superiority of proposed algorithm. Our method also yields better results than the popular median filtering techniques used for denoising impulse noise.

I. INTRODUCTION

Image denoising is a classical problem as images are often corrupted by different kinds of noises - monochromatic noise [1], white Gaussian noise [2], non-white noise [3], impulse noise [4] etc. This work addresses the problem of reducing impulse noise from gray-scale images. The main source of impulse noise is the random fluctuations in the power supply of image capturing device. Impulse noise can be classified as fixed valued impulse noise or random valued impulse noise. Fixed valued impulse noise is also called salt and pepper noise in which each noisy pixel have either maximum or minimum intensity value. Random valued impulse noise corrupted pixels can take any random value within the intensity range. The noise model can be expressed as follows:

\[ y = x + n \] (1)

where \( x \) is the original (uncorrupted) image in vector form obtained by vertical concatenation, \( n \) is noise and \( y \) is the noise corrupted image. The problem is to estimate \( x \) given \( y \) and knowledge about the noise model. There are two broad approaches to solve this problem. The first approach is to use median filtering and its variants such as [5], [6], [7], [8]. The other approach is to exploit the sparsity of the image in some transform domain and formulate denoising as an optimization problem. The focus of this work is on optimization based approach to denoising.

The synthesis and analysis equations for orthogonal sparsifying transform domain can be expressed as \( x = A\alpha \) and \( \alpha = A^T x \) respectively. where \( A \) is the orthogonal sparsifying transform and \( \alpha \) denotes the sparse transform coefficients. Incorporating the sparsity inducing transform into (1) leads to:

\[ y = A\alpha + n \]

One can solve for the sparse transform coefficients by minimizing the following equation:

\[
\min_{\alpha} \| y - A\alpha \|_q^p + \lambda \| \alpha \|_p \quad 0 < p, q \leq 1
\] (2)

usually for Gaussian noise, \( q = 2 \) is employed, whereas the value of \( p \) lies between 0 and 1 and \( \lambda \) is the regularization parameter. But since impulse noise is sparse therefore one needs to use \( q \) between 0 and 1 [9]. This formulation (2) is referred as synthesis prior (SP) formulation. If the sparsifying domain is not orthogonal then one can frame an analysis prior problem. Total variation (TV) [10] based denoising, which is basically finite difference transform, is one classic example of analysis prior. The analysis prior holds for all linear transforms. The image recovery problem using analysis prior (AP) formulation can be expressed as follows:

\[
\min_{x} \| y - x \|_q^p + \lambda \| D_h x \|_p^p + \lambda \| D_v x \|_p^p \quad 0 < p, q \leq 1
\] (3)

where \( D_h \) and \( D_v \) are horizontal and vertical finite difference operators. Problems like (2) and (3) were previously solved using iterated reweighted norm (IRN) minimization techniques [4], [11], [12]. However, such IRN schemes only converge to the solution of problems (2), (3) asymptotically. In this work, we propose a split-Bregman type algorithm to solve these problems exactly. Prior studies have used Bregman iteration [13] and split-Bregman [14] methods to successfully solve \( \ell_1 \)-norm regularized least squares problems. We solve a more general problem.

In this work our main contribution is to derive analysis prior and synthesis prior optimization problems to solve general \( \ell_p \)-norm minimization problem with \( \ell_q \)-norm regularization constraint. We have applied our proposed algorithm for the denoising single band of color, multispectral, and hyperspectral images corrupted by impulse noise.

II. PROPOSED ALGORITHMS

In this section, we derive algorithms to solve synthesis prior (2) and analysis prior (3) problems described earlier. We have utilized split-Bregman [14] technique which is similar to augmented-Lagrangian (AL)/alternating direction method of multiplier (ADMM). This technique allow us to break the problem into relatively easy to solve subproblems. This technique is also helpful when there are multiple regularization parameters.
A. Synthesis Prior Algorithm

As mentioned before, the problem is to solve (2). We repeat it here for the sake of convenience.

\[
\min_{\alpha} \|y - A\alpha\|_q^2 + \lambda \|\alpha\|_p^p
\]

Since the variable \(\alpha\) is not separable therefore we substitute: \(u = y - A\alpha\) and \(v = \alpha\) then (4) can be rewritten as constrained optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \mu_1 \|u - y + A\alpha\|_2^2 \\
\text{subject to} & \quad u = y - A\alpha \\
& \quad v = \alpha
\end{align*}
\]

The above constrained optimization problem can be expressed as unconstrained optimization problem using weak penalty function as follows:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \mu_1 \|u - y + A\alpha - b_1^k\|_2^2 \\
& + \mu_2 \|v - \alpha - b_2^k\|_2^2
\end{align*}
\]

where \(\mu_1\) and \(\mu_2\) are the regularization parameters. Since there are multiple regularization terms therefore split Bregman [14] approach can be applied to solve this problem. Thus the above unconstrained problem can be expressed as:

\[
f(\alpha, u, v) = \min_{\alpha, u, v} \|u\|_q^2 + \lambda \|v\|_p^p + \mu_1 \|u - y + A\alpha - b_1^k\|_2^2 \\
+ \mu_2 \|v - \alpha - b_2^k\|_2^2
\]

where \(b_1^k\) and \(b_2^k\) are the Bregman variables. The algorithm can be written as an unconstrained problem using weak penalty function as:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \lambda \|w\|_p^p + \mu_1 \|u - y + x\|_2^2 \\
& + \mu_2 \|v - D_h x\|_2^2 + \mu_2 \|w - D_v x\|_2^2
\end{align*}
\]

Using split-Bregman approach, above problem can be expressed as:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \lambda \|w\|_p^p + \mu_1 \|u - y + x\|_2^2 \\
& + \mu_2 \|v - D_h x\|_2^2 + \mu_2 \|w - D_v x\|_2^2 \\
\end{align*}
\]

which can be solved using general soft-thresholding algorithm [15], whose solution is given by:

\[
\hat{x} = \text{SoftTh}(z, \lambda, p) = \text{sign}(z) \times \max\{0, |z| - \lambda p|z|^{p-1}\} \quad (7)
\]

Algorithm 1 summarizes the steps of the proposed algorithm.

Algorithm 1 Synthesis Prior (SP) Algorithm

1: input: \(A, y, \lambda, \mu_1, \mu_2\), MaxIter.
2: output: \(\hat{x}\), denoised image.
3: for \(k = 1\) to MaxIter do
4: \(\alpha^{k+1} = \alpha^k\) from (5)
5: \(u^{k+1} = \text{SoftTh}(y - A\alpha^k + b_1^k, \mu_1, q)\) from (7)
6: \(v^{k+1} = \text{SoftTh}(\alpha^k + b_2^k, \mu_2, p)\) from (7)
7: \(b_1^{k+1} = b_1^k - u^{k+1} + y - A\alpha^{k+1}\)
8: \(b_2^{k+1} = b_2^k + \alpha^{k+1} - v^{k+1}\)
9: end for
10: \(\hat{x} = A\alpha^{k+1}\)

B. Analysis Prior Algorithm

The analysis prior formulation as described earlier can be expressed as:

\[
\arg\min_x \|y - x\|_q^2 + \lambda \|D_h x\|_p^p + \lambda \|D_v x\|_p^p \quad (8)
\]

As before, we make substitutions and recast (8) as follows:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \lambda \|w\|_p^p + \mu_1 \|u - y + x\|_2^2 \\
& + \mu_2 \|v - D_h x\|_2^2 + \mu_2 \|w - D_v x\|_2^2
\end{align*}
\]

which can be written as an unconstrained problem using weak penalty function as:

\[
\begin{align*}
\text{minimize} & \quad \|u\|_q^2 + \lambda \|v\|_p^p + \lambda \|w\|_p^p + \mu_1 \|u - y + x\|_2^2 \\
& + \mu_2 \|v - D_h x\|_2^2 + \mu_2 \|w - D_v x\|_2^2
\end{align*}
\]

These problems are \(\ell_p\)-norm minimization problems which are similar to the following problem

\[
\arg\min_x \|z - x\|_2^2 + \lambda \|x\|_p^p \quad (6)
\]

Problems (P2) and (P3) can be re-written as:

\[
\begin{align*}
\text{P2:} & \quad \arg\min_u \|(y - A\alpha + b_1^k) - u\|_2^2 + \frac{1}{\mu_1} \|u\|_q^2 \\
\text{P3:} & \quad \arg\min_v \|(\alpha + b_2^k) - v\|_2^2 + \frac{\lambda}{\mu_2} \|v\|_p^p
\end{align*}
\]

where \(b_1, b_2, b_3\) are Bregman variables updated as follows:

\[
\begin{align*}
& b_1^{k+1} = b_1^k + y - u - x \\
& b_2^{k+1} = b_2^k + D_h x - v \\
& b_3^{k+1} = b_3^k + D_v x - w
\end{align*}
\]

The above problem can be split to solve for four separable variables \((x, u, v, w)\) as follows:

\[
\begin{align*}
\text{P4:} & \quad \arg\min_x \mu_1 \|u - y + x - b_1^k\|_2^2 + \mu_2 \|v - D_h x - b_2^k\|_2^2 \\
& + \mu_2 \|w - D_v x - b_3^k\|_2^2
\end{align*}
\]

\[
\begin{align*}
\text{P5:} & \quad \arg\min_u \|u\|_q^2 + \mu_1 \|u - y + x - b_1^k\|_2^2 \\
\text{P6:} & \quad \arg\min_v \|v\|_p^p + \mu_2 \|v - D_h x - b_2^k\|_2^2 \\
\text{P7:} & \quad \arg\min_w \|w\|_p^p + \mu_2 \|w - D_v x - b_3^k\|_2^2
\end{align*}
\]
In this case also problem $P4$ can be differentiated to get:

\[(\mu_1 I + \mu_2 (D_h^T D_h + D_v^T D_v)) x = \mu_1 (y - u + b_1^k) + \mu_2 (D_h^T (v - b_2^k) + D_v^T (w - b_3^k)) \]  

this linear system of equations is large and sparse which can be solved using a few iterations of least square solvers such as LSQR [16]. Problems $P5$, $P6$, and $P7$ have form similar to (6) and can be solved using (7) as described in previous section. Algorithm 2 summarizes the analysis prior algorithm.

**Algorithm 2** Analysis Prior (AP) Algorithm

1: input: A, y, \(\lambda\), \(\mu_1\), \(\mu_2\), MaxIter.
2: output: \(\hat{x}\), denoised image.
3: for \(k = 1\) to MaxIter do
4: \(x^{k+1} = x^k\) from (9)
5: \(w^{k+1} = \text{SoftTh} (y - x^{k+1} + b_1^k, \frac{1}{\mu_1}, q)\)
6: \(v^{k+1} = \text{SoftTh} (D_h x^{k+1} + b_2^k, \frac{1}{\mu_2}, p)\)
7: \(u^{k+1} = \text{SoftTh} (D_v x^{k+1} + b_3^k, \frac{1}{\mu_2}, p)\)
8: \(b_1^{k+1} = b_1^k + y - u^{k+1} - x^{k+1}\)
9: \(b_2^{k+1} = b_2^k + D_h x^{k+1} - v^{k+1}\)
10: \(b_3^{k+1} = b_3^k + D_v x^{k+1} - w^{k+1}\)
11: end for
12: \(\hat{x} = x^{k+1}\).

### III. Experiments and Results

Experiments were performed on images from color image dataset [17], multispectral dataset [18] and hyperspectral dataset [19]. Kodak dataset [17] have 24 color images each of which is a 768 x 512 x 3. The multispectral dataset have 31 images with each image having dimensions 512 x 512 x 31 and all the bands are in spectral range of 400 nm to 700 nm. The hyperspectral image of Washington DC mall [19] is of Hyperspectral Digital Imagery Collection Experiment (HYDICE) sensor having 1ns spatial resolution and 10-nm band spacing covering spectral range of 400-2500 nm. We considered a patch of size 160 x 160 x 191 from WDC image for experiments. Single band was considered for performing experiments from these datasets.

Noisy images were generated by corrupting original images from 10% to 70% impulse noise. Both salt and pepper noise and random valued impulse noise were considered for performing experiments. Daubechies wavelet transform (db8) with filter length 8 was used as sparsifying transform domain in the synthesis prior (SP) algorithm whereas total variation was used in the analysis prior (AP) algorithm. Since impulse noise is sparse therefore value of \(p\) and \(q\) were both kept one. Empirically we had found values of parameters \(\lambda\), \(\mu_1\) and \(\mu_2\) to be 1, 1 and 1 respectively for the SP algorithm whereas 1, 2, and 1 respectively for AP algorithm. Bregman variables \(b_1^0\) and \(b_2^0\) and \(b_3^0\) were initialized to zero in both the algorithms. We used Peak Signal to Noise Ratio (PSNR) and structural similarity index (SSIM) [20] to quantitatively measure the quality of denoised images.

We have compared our proposed algorithms with [11] in which Iterative Reweighted Norm (IRN) based algorithm have been proposed to minimize total variation functional. As discussed in [11], five loops of IRN algorithm were used throughout all experiments since solution does not improve beyond that. Comparative study with standard median filtering (MF) was also carried out. A window size of 5 x 5 was used when noise levels was above 40%.

Figures 1 and 2 shows convergence graph for synthesis prior and analysis prior algorithms. The objective function decreases with number of iterations and after few iterations it becomes almost constant. The decrease is not linear as expected since the algorithms are based on split-Bregman technique.

Table I shows experimental results on images from Kodak dataset. Experiments were done from 10% to 50% noise though table shows results for 30% salt and pepper and random impulse noise. Peak signal to noise ratio (PSNR) and structural similarity index (SSIM) [20] were calculated on all 24 images for IRN, proposed SP and AP algorithms. Column Noisy shows PSNR of noisy images. Average SSIM of noisy images was 0.01. Although average PSNR for SP algorithm is low as compared to IRN algorithm but SSIM index is high and AP algorithm outperforms IRN algorithm for both PSNR and SSIM.

Table II shows experimental results on single band of
### Table I

**Comparison of PSNR and SSIM values on images from Kodak dataset [17].**

<table>
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<th>Noisy SSIM</th>
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### Table II

**Comparison of PSNR and SSIM values on images from multispectral dataset [18].**

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Avg: 25.36, 30.42, 34.39, 0.88, 0.92, 0.95, 0.97
Fig. 3. Visual comparison of proposed algorithms SP and AP with IRN algorithm on WDC mall, Lena, and Balloon Images. Columns 1, 2 and 3 shows recovered images for 10%, 30% and 50% noise respectively.
Fig. 4. Visual comparison of proposed algorithms with MF and IRN for 70% salt and pepper noise on Balloon image. Recovery with proposed AP algorithm is better as compared to others.

### TABLE III

**RESULTS FOR 70% SALT AND PEPPER NOISE ON IMAGES FROM MULTISPECTRAL DATASET**

<table>
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<th>AP</th>
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multispectral images for salt and pepper noise as well as random impulse noise. Experiments were done on all 31 images with all noise levels but here results are shown on 24 images for 50% noise. Average PSNR value for noise corrupted images was 6.77 dB and 8.96 dB for salt and pepper noise and random impulse noise respectively while average SSIM value was zero for both kind of noise. PSNR and SSIM values indicate that both SP and AP algorithms perform better than IRN algorithm as well as median filter.

Figure 3 visually compares reconstruction quality of AP and SP algorithms with IRN algorithm. It is visually clear that reconstruction quality for hyperspectral and multispectral image is better for proposed algorithms and competitive on Lena image. Figure 4 compare recovery results for 70% salt and pepper noise on Ballon image. Clearly reconstruction quality of proposed AP algorithm is better than other algorithms compared against.

IV. CONCLUSIONS

Proposed synthesis and analysis prior algorithms are able to reduce both salt and pepper noise and random value impulse noise from color, multispectral, and hyperspectral images. The algorithms can be applied on each band individually to denoise all the bands. Quantitative and qualitative results suggests that the algorithms are competitive or better than existing algorithm in terms of PSNR, Structural similarity and visual quality. The capability of the current algorithm is limited to each spectral band separately. It does not account for inter-band correlation. In future, we look forward to extend these denoising methods for multiple bands by taking into account the spectral correlations.

Our Matlab implementation of both the algorithms are available from [21] for the sake of reproducible research.

V. ACKNOWLEDGMENTS

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REFERENCES


