MIXED GAUSSIAN AND IMPULSE DENOISING OF HYPERSPECTRAL IMAGES

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ABSTRACT

Hyperspectral image denoising is an important preprocessing step in the analysis of hyperspectral images in several applicaitons domains. These images often gets corrupted by various kinds of noise during acquisition process. There are several studies on reducing Gaussian noise from hyperspectral images. This work addresses the problem of reducing mixed noise from hyperspectral images; in particular a mixture of Gaussian and impulse noise has been considered. The proposed image acquisition model explicitly accounts for both Gaussian and impulse noise as additive noise. This mixed noise reduction problem has been formulated as synthesis prior optimization problem which exploits inherent spatio-spectral correlation present in hyperspectral images. Split-Bregman based approach has been utilized to solve resulting optimization problem. Experiements were conducted using both synthetic noise as well as real noisy hyperspectral images. Experimental results have been quantified using peak signal to noise ratio (PSNR) and structural similarity index (SSIM). A comparative study with an existing low-rank based image denoising approaches has also been carried out. Both quantitative and qualitative results suggest the superiority of proposed approach.

***Index Terms*—** Gaussian noise, Impulse Noise, Hyperspectral Images, Split-Bregman

1. INTRODUCTION

Images captured over hundreds of bands of electromagnetic spectra ranging from around 400 nm to 2500 nm are generally termed as hyperspectral images. These images are useful in various application domains such as agriculture, forensics, resource management, environmental monitoring etc. Most of the applications requires denoising as a pre-processing step. Images are corrupted by noise due to several reasons including fluctuations in power supply, dark current, and non-uniformity of detector response etc. The corruption model can be represented as a mixture of Gaussian and Impulse noise [1, 2].

Hyperspectral denoising for Gaussian noise is a well studied problem [3, 4, 5]. The approach [3] applies principle component analysis on each band and then do wavelet shrinkage only on the low energy principle component bands and keep top few principle component bands intact. The work [4] utilizes adaptive total variation based approach to denoise the hyperspectral image. The low rank structure of hyperspectral images has been exploited in [5]. Hyperspectral unmixing based denoising approach has been proposed in [6]. To the best of our knowledge there is no prior work on removing impulse noise from hyperspectral images (though there is plethora of work on impulse denoising of grayscale images). In this paper we propose to remove both Gaussian and impulse noise from a hyperspectral image. This is a mixed noise reduction problem. There are studies such as [1, 2] which consider mixed noise reduction from gray scale images however this work address the problem of denoising hyperspectral images.

We exploit the spatio-spectral correlation of the hyperspectral datacube for denoising. The formulation is loosely based on the concept of Kronecker compressed sensing [7]. The spatial correlation exists since in most satellite images nearby objects or areas are similart to each other due to which neighboring pixels have quite similar intensity value. The spectral correlation exists in hyperspectral images since the band gap in captured bands is very low. The band gap is between neighboring bands is generally in range of 5 nm to 10 nm. We have utilized 2D discrete cosine transform (DCT) to sparsify the image along spatial dimension and 1D-DCT to sparsify the image along spectral dimension. We have formulated the mixed (additive Gaussian and Impulse) denoising problem as a synthesis prior problem and solved the resulting optimization problem using split-Bregman approach [8]. We have also compared our technique with an existing low rank matrix recovery (LRMR) [5] based algorithm. Peak signal to noise ratio (PSNR) and structural similarity index (SSIM) [9] are used to quantify the denoising results. Experimental results indicate that our proposed method is about 4.5 dB better in PSNR and 40% better in SSIM compared to existing technique [5] for a mixture of Gaussian and impulse noise.

2. PROBLEM FORMULATION

Define $y = vec(Y)$ as vector representation of any 2D matrix obtained by vertical stacking of columns of matrix Y.We use small letters for vectors and capital letters for matrices. A hyperspectral data cube of dimension $m \times n \times d$ having d spectral

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bands in it can be represented as $X = \begin{bmatrix} x_1 & x_2 & \dots & x_d \end{bmatrix}$ where each $x_i \in \mathbb{R}^{mn\times 1}$ is a spectral band obtained by vertical concatenation. Using these notations, image acquisition model in the presence of Gaussian and impulse noise can be expressed as:

$$Y = X + N_1 + N_2,$$

where $X \in \mathbb{R}^{mn\times d}$ is original image, Y is noise corrupted image, N_1 is Gaussian noise, and N_2 is impulse noise. Each columns of X is an image and hence can be sparsely represented in transforms like wavelet or DCT. Since different bands in a hyperspectral image are spectrally correlated, the variation along the rows of X can also be sparsely represented. The spatio-spectral correlation can be jointly exploited by representing the datacube X as a sparse signal Z in a combined transform domain such that $Z = D_1XD_2$. Here D_1 is a 2-D transform applied along spatial dimension, and D_2 is 1-D transform applied on spectral dimension. If the sparsifying transform domain is orthogonal or tight-frame (wavelet, DCT etc.) then we can express it in the synthesis prior form as $X = D_1^T Z D_2^T$. Using the synthesis prior (SP) formulation, the denoising problem can be framed as:

$$\min_{Z,N_2} \|Z\|_1 + \|N_2\|_1 + \lambda\|Y - D_1^T Z D_2^T - N_2\|_F^2 \quad (1)$$

where λ is a regularization parameter. Here ℓ_1-norm of Z is minimized since Z is a sparse representation of the hyperspectral image in the transform domain, ℓ_1-norm of N_2 is minimized because N_2 is representing impulse noise which is a sparse in nature. Since N_1 is Gaussian noise and $N_1 = Y - X - N_2$, therefore we are minimizing Frobenius norm of $Y - D_1^T Z D_2^T - N_2$ to reduce Gaussian noise. Although researchers have proposed generic noise removal techniques [5, 3], such an explicit formulation for jointly denoising Gaussian and impulse noise has not been attempted before. This is a new problem and we are not aware of any efficient algorithm that can solve the aforesaid problem. Therefore in the next section we describe how to solve this problem using split-Bregman [8] based approach.

3. PROPOSED ALGORITHMS

This section discusses how to solve (1) using split-Bregman approach. This approach had been very successful in solving multiple penalty optimization problems [10, 11]. We repeat synthesis prior problem for the sake of convenience.

$$\min_{Z,N_2} \|Z\|_1 + \|N_2\|_1 + \lambda\|Y - D_1^T Z D_2^T - N_2\|_F^2$$

Since the variable Z is not separable, we substitute $P = Z$ and $Q = N_2$ such that above problem can be rewritten as an unconstrained optimization problem :

$$\begin{aligned}\underset{Z,P,Q,N_2}{\text{minimize}}\ & \|P\|_1 + \|Q\|_1 + \frac{\lambda}{2}\|Y - D_1^T Z D_2^T - N_2\|_F^2 \\ & + \frac{\mu_1}{2}\|P - Z - B_1\|_F^2 + \frac{\mu_2}{2}\|Q - N_2 - B_2\|_F^2\end{aligned}$$

where λ, μ_1, μ_2 are regularization parameters and B_1, B_2 are Bregman variables. Since there are multiple regularization terms, we intent to follow the split-Bregman [8] approach which can be applied to solve this problem. Thus the above unconstrained problem can be split into following sub-problems as:

$$\text{P1}: \quad \min_Z \frac{\lambda}{2}\|Y - D_1^T Z D_2^T - N_2\|_F^2 + \frac{\mu_1}{2}\|P - Z - B_1\|_F^2$$

$$\text{P2}: \quad \min_P \|P\|_1 + \frac{\mu_1}{2}\|P - Z - B_1\|_F^2$$

$$\text{P3}: \quad \min_Q \|Q\|_1 + \frac{\mu_2}{2}\|Q - N_2 - B_2\|_F^2$$

$$\text{P4}: \quad \min_{N_2} \frac{\lambda}{2}\|Y - D_1^T Z D_2^T - N_2\|_F^2 + \frac{\mu_2}{2}\|Q - N_2 - B_2\|_F^2$$

Here subproblems P1 and P4 are least square problems with analytic solutions:

$$Z = \frac{1}{\lambda + \mu_1}\left(D^T(Y - N_2) + \mu_1(P - B_1)\right)$$

$$N_2 = \frac{1}{\lambda + \mu_2}\left(\lambda(Y - DZ) + \mu_2(Q - B_2)\right)$$

Subproblems P2 and P3 are of the form :

$$\arg\min_x \|y - x\|_2^2 + \lambda\|x\|_1$$

which can be solved by using soft-thresholding [12] $\mathrm{SoftTh}(y, \lambda)$ operation:

$$\hat{x} = \mathrm{sign}(y) \times \max\left\{0, |y| - \frac{\lambda}{2}\right\} \quad (2)$$

Algorithm 1 summaries the steps of the proposed synthesis prior approach.

Algorithm 1 Synthesis Prior (SP) Algorithm

1: Input: $D_1, D_2, D = D_2 \otimes D_1^T, Y, \lambda, \mu_1, \mu_2$, Iter.
2: Output: $\hat{X}$, denoised image.
3: **for** $k = 1$ to Iter **do**
4: $\quad Z^{k+1} = \frac{1}{\lambda+\mu_1}\left(D^T(Y - N_2^k) + \mu_1(P^k - B_1^k)\right)$
5: $\quad P^{k+1} = \mathrm{SoftTh}\left(Z^{k+1} + B_1^k, \frac{2}{\mu_1}\right)$
6: $\quad Q^{k+1} = \mathrm{SoftTh}\left(N_2^{k+1} + B_2^k, \frac{2}{\mu_2}\right)$
7: $\quad N_2^{k+1} = \frac{\lambda}{\lambda+\mu_2}(Y - DZ^{k+1}) + \frac{\mu_2}{\lambda+\mu_2}(Q^{k+1} - B_2^k)$
8: $\quad B_1^{k+1} = B_1^k - P^{k+1} + Z^{k+1}$
9: $\quad B_2^{k+1} = B_2^k + N_2^{k+1} - Q^{k+1}$
10: **end for**
11: $\hat{X} = D_1^T Z^{k+1} D_2^T$

4. EXPERIMENTS AND RESULTS

Experiments were performed using two hyperspectral datasets. The first hyperspectral image was of Washington DC (WDC) mall [13] from Hyperspectral Digital Imagery Collection Experiment (HYDICE) sensor having $1m$ spatial resolution and 10-nm band spacing covering spectral range of 400-2500 nm. We considered a patch of size $256 \times 256 \times 191$ from *WDC* image for experiments. Second image was of Gulf of Mexico area [14] from SpecTIR having $2m$-spatial resolution and 5-nm band spacing covering spectral range of 395-2450-nm. A patch of size $256 \times 256 \times 360$ was considered for performing experiments.

The proposed algorithm is compared with an existing low rank matrix recovery based algorithm LRMR [5]; this technique was developed to solve generic hyperspectral denoising problems. It explores low-rank nature of a hyperspectral image for denoising. During experiments we considered images from all the spectral bands. In the previous studies like LRMR [5] extremely noise bands were not considered in experiments; skipping over noisy bands defeats the purpose of denoising.

We utilized 3D-DCT as sparsifying transform in the proposed synthesis prior algorithm, i.e. 2D-DCT to sparsify each spectral band image and 1D-DCT to sparsify across the spectral bands. All the unknown variables (Z,N_2,P, Q, B_1, B_2) required by our algorithm were initialized to zero. All three parameters ($\lambda = 0.5, \mu_1 = 1, \mu_2 = 1$) were found empirically. Parameters for the LRMR algorithm (rank=10 and sparsity=4000) were set to yield the best results as described in the paper [5].

Experiments were performed with both synthetically added noise as well as real noisy bands in raw hyperspectral images. For the case of synthetic noise, all the bands were corrupted by mixture of Gaussian noise with standard deviation 20 and 30 as well as 10% to 50% impulse noise. Table 1 summarizes comparison of PSNR and SSIM values for the proposed algorithm(SP) and benchmark technique LRMR. It can be observed from Table 1 that in all experiments proposed technique outperform existing LRMR approach.

Figure 1 visually compare quality of restoration of proposed method with the existing LRMR technique for synthetic noise case. Gaussian noise of standard deviation 20 and 30% impulse noise was added to the image. It can be observed from the denoised images that proposed approach has significantly reduced the noise whereas with LRMR approach some smoothing effect is visible. The images are shown in false color composite of bands 20, 90, 190 for WDC image. Histogram equilizatiaon was applied on all the images for visual display only.

Experiments were also performed with real noisy hyperspectral images. Figure 2 shows denoised Gulf image with bands 40, 130, 220 in false color composite. It can be observed tha original Gulf image is a noisy having different kinds of noise. This noisy image was further contaminated by Gaussian noise of standard deviation 30 and 30% impulse nosie. In this case also proposed approach gives more visually appealing results compared to LRMR approach. Visually it can be observed from Gulf image that not only mixed noise but also vertical line strips present in original image have been reduced by our proposed technique. Line striping problem in satellite images occurs when some sensors are out of radiometric calibration.

Table 1: Comparison of PSNR and SSIM values for the existing LRMR and proposed synthesis prior (SP) algorithm.

WDC image						
	PSNR(dB)			SSIM		
Noise	Noisy	LRMR	SP	Noisy	LRMR	SP
20+10%	14.36	23.41	**25.08**	0.45	0.71	**0.8**
20+20%	11.71	20.68	**23.10**	0.32	0.6	**0.78**
20+30%	10.07	19.43	**22.71**	0.23	0.53	**0.76**
20+40%	8.89	18.11	**22.08**	0.17	0.44	**0.72**
20+50%	7.96	16.64	**21.16**	0.13	0.37	**0.66**
Gulf of Mexico image						
30+10%	13.20	21.94	**24.02**	0.25	0.55	**0.72**
30+20%	10.69	19.51	**23.49**	0.17	0.43	**0.70**
30+30%	9.11	17.26	**22.80**	0.12	0.36	**0.67**
30+40%	7.96	16.34	**22.18**	0.09	0.32	**0.63**
30+50%	7.04	14.74	**21.24**	0.07	0.26	**0.55**

5. CONCLUSIONS

In this work, we proposed an algorithm to reduce mixed Gaussian and impulse noise from hyperspectral images by exploring inherent spatial and spectral correlation present in these images. Proposed synthesis prior algorithm is able to achieve higher PSNR and SSIM values compared to existing algorithm. The visual quality of restored hyperspectral image by proposed algorithm is better than the sate-of-the-art algorithm. As a future work, we are working to simultaneously explore sparsity and low rank nature of hyperspectral images for mixed noise reduction problem.

6. REFERENCES

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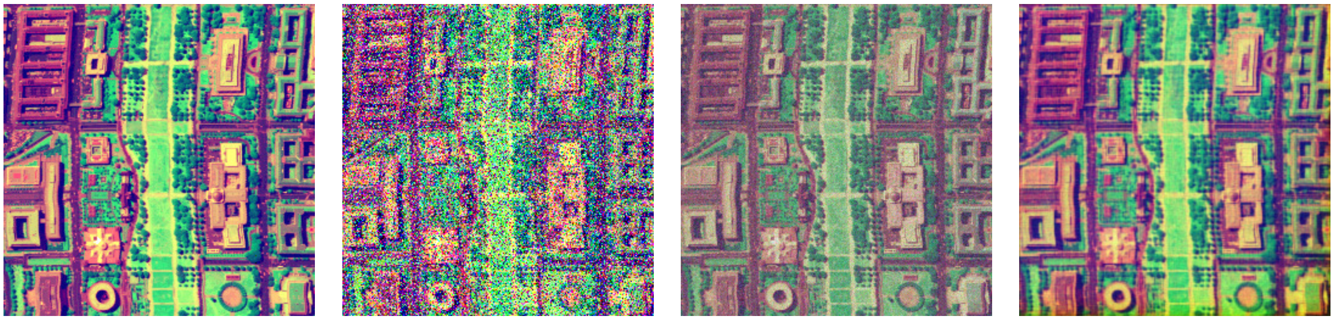
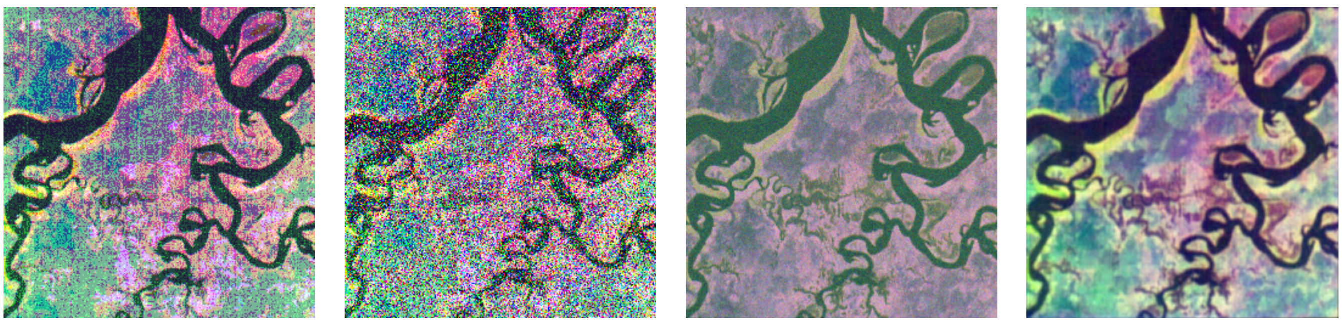


(a) Original WDC image (b) Noisy, PSNR=10.07 dB (c) LRMR, PSNR=19.4 dB (d) SP, PSNR=22.71 dB

Fig. 1: Results on WDC mall image for mixed Gaussian noise of std. dev. 20 and 30% impulse noise.



(a) Original Gulf image (b) Noisy, PSNR=9.11 dB (c) LRMR, PSNR=17.2 dB (d) SP, PSNR=22.80 dB

Fig. 2: Results on Gulf of Mexico image for mixed Gaussian noise of std. dev. 30 and 30% impulse noise.

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